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Conformal higher spin currents in any dimension and AdS/CFT correspondence

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Abstract: The full list of conserved conformal higher spin currents built from massless scalar and spinor fields is presented. It is shown that, analogously to the relationship between usual conformal and AdS symmetries, the set of the conformal higher spin symmetry parameters associated with the conformal conserved currents in d dimensions is in the one-to-one correspondence with the result of the dimensional reduction of the usual (i.e. non-conformal) higher spin symmetry parameters in $d+1$ dimensions.

Keywords: AdS-CFT Correspondance, Conformal and W Symmetry.

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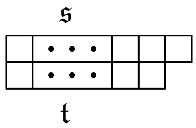
Contents

1. Introduction

Usual lower spin conserved currents admit a natural extension to the higher spin case in any space-time dimension. Historically first examples of the higher spin currents were known as zilch currents [1]. More examples were given in [2, 3]. The full list of conserved higher spin currents, including those containing explicit dependence on the spacetime coordinates, was given recently in [4] where it was shown in particular that various types of conserved currents $J^n{}_{;a(\mathfrak{t}),b(\mathfrak{s})}$ associated with the integer spin $s = \mathfrak{s}+1$ are vector fields (index n) taking values in the representations of the Lorentz group $\mathrm{SO}(d-1,1)$ described by the traceless two-row Young diagrams



(1.1)

with $0 \leq \mathfrak{t} \leq \mathfrak{s}$. This means that the tensor $J^n{}_{;a(\mathfrak{t}),b(\mathfrak{s})}$ is symmetric in the indices b and a separately[1] ($a, b, n = 0, \dots, d-1$, where d is the space-time dimension), obeys the antisymmetry property

$$J^n{}_{;a(\mathfrak{t}-1)b(1),\, b(\mathfrak{s})} = 0 \qquad (1.2)$$

and has zero contractions of indices a and/or b.

[1]Following to [5], we use notation $a(\mathfrak{t})$ for multiindices $a_1 a_2 \dots a_{\mathfrak{t}}$ subject to symmetrization (by symmetrization we mean the appropriate projector, i.e. symmetrization of a symmetric tensor leaves it unchanged). Multiindex $a(\mathfrak{t})a(\mathfrak{s})$ is equivalent to $a(\mathfrak{t}+\mathfrak{s})$. We use notation $A^{a(\mathfrak{t})}B_{a(\mathfrak{t})}$ for $A^{a_1 \dots a_{\mathfrak{t}}}B_{a_1 \dots a_{\mathfrak{t}}}$ with symmetrized tensors A and B. The tensor indices are raised and lowered by the flat Minkowski metric η_{ab}.

It is enough to require

$$J^{n}{}_{;a(\mathfrak{t}),b(\mathfrak{s}-2)c}{}^{c} = 0 \tag{1.3}$$

since from (1.2) it follows then that any other contraction of indices a and/or b is zero.

It is convenient to consider the currents

$$J^n(\xi) = J^{n}{}_{;}{}^{a(\mathfrak{t})}{}_{,}{}^{b(\mathfrak{s})}\xi_{a(\mathfrak{t}),b(\mathfrak{s})}\,, \tag{1.4}$$

where $\xi_{a(\mathfrak{t}),b(\mathfrak{s})}$ are some constant parameters which transform under the irreducible representation of Lorentz group corresponding to the traceless two-row Young diagram. The role of the parameter $\xi_{a(\mathfrak{t}),b(\mathfrak{s})}$ is that it projects the current to the appropriate irreducible representation of the Lorentz group.

The current $J^n(\xi)$ conserves if

$$\partial_n J^n(\xi) = 0\,. \tag{1.5}$$

In the cases discussed below, a conserved current corresponding to the one-row Young diagram admits a representation

$$J^{n}{}_{;\,,}{}^{a(\mathfrak{s})}\xi_{a(\mathfrak{s})} = T^{n\,a(\mathfrak{s})}\xi_{a(\mathfrak{s})}\,, \tag{1.6}$$

where $T^{a(\mathfrak{s}+1)}$ is a totally symmetric conserved current

$$\partial_n T^{na(\mathfrak{s})} = 0\,. \tag{1.7}$$

Given conserved current $T^{n(\mathfrak{s}+1)}$, it allows one to construct the set of integer spin conserved currents [4],

$$J^n(\xi) = T^{nl(\mathfrak{s})}x^{m(\mathfrak{t})}\xi_{m(\mathfrak{t}),l(\mathfrak{s})}\,, \tag{1.8}$$

where we use the shorthand notation

$$x^{m(\mathfrak{s})} = x^{m_1}\dots x^{m_{\mathfrak{s}}}\,. \tag{1.9}$$

These generalize the usual spin one current T^n, stress tensor T^{nm} and angular momentum current

$$M^{n\;b,a} = T^{na}x^b - T^{nb}x^a \tag{1.10}$$

to an arbitrary integer spin. A form of the symmetric current $T^{a(s)}$ depends on a particular matter system. For scalar and spinor fields some examples are given in [3, 4] for any spin. Note that the current $T^{a(s)}$ is not required to be traceless in this construction. However, the terms with double traces in $T^{a(s)}$ do not contribute to (1.8) because $\xi_{a(\mathfrak{t}),b(\mathfrak{s})}$ is traceless. The situation with supercurrents is analogous [4].

It is well known that if the stress tensor is traceless, this implies a larger conformal symmetry. Indeed, in that case the currents associated with the dilatation (scale transformation)

$$T_1^n = T^{nk}x_k \tag{1.11}$$

and special conformal transformations

$$K_1^{na} = T^{na}x^2 - 2T_1^n x^a\,, \qquad (x^2 = \eta_{ab}x^a x^b) \tag{1.12}$$

conserve. The currents (1.10), (1.11) and (1.12) and T^{na} itself exhaust all those currents that can be constructed from the traceless stress tensor T^{na}.

It was conjectured in [4] that a similar phenomenon takes place for all spins, implying larger conformal higher spin symmetries for the case of traceless tensors (1.6). The aim of this paper is to give the full list of conserved conformal higher spin currents that can be built from scalar and spinor fields. To elucidate a general pattern we first consider the simplest nontrivial example of the spin three current.

2. Spin three example

Let the symmetric tensor T^{abc} be traceless and conserved

$$\begin{aligned} \partial_a T^{abc} &= 0\,, \\ T^{ab}{}_b &= 0\,. \end{aligned} \tag{2.1}$$

First, one observes that the traceless tensor

$$T_1^{n\ a} = T^{nak}x_k \tag{2.2}$$

conserves. It allows one to construct conserved currents analogous to (1.10), (1.11) and (1.12):

$$\begin{aligned} T_2^n &= T_1^{n\ k}x_k \\ M_1^{n\ b,a} &= T_1^{n\ a}x^b - T_1^{n\ b}x^a\,, \end{aligned} \tag{2.3}$$

and

$$K_1^{n\ a} = T_1^{n\ a}x^2 - 2T_2^n x^a\,, \qquad x^2 \equiv x_b x^b\,. \tag{2.4}$$

In addition, one can check that the following currents conserve

$$K_2^{n\ a_1a_2} = T^{na_1a_2}(x^2)^2 - 2\left(T_1^{n\ a_1}x^{a_2} + T_1^{n\ a_2}x^{a_1}\right)x^2 + 4T_2^n x^{a_1}x^{a_2}\,, \tag{2.5}$$

$$K_1^{n\ a_1a_2} = T^{na_1a_2}x^2 - T_1^{n\ a_1}x^{a_2} - T_1^{n\ a_2}x^{a_1} + \frac{2}{d}T_2^n\eta^{a_1a_2}\,, \tag{2.6}$$

$$\begin{aligned} M^{n\ b,a_1a_2} &= 2T^{na_1a_2}x^b - T^{na_1b}x^{a_2} - T^{nba_2}x^{a_1} + \\ &+ \frac{1}{d-1}\left(2\eta^{a_1a_2}T_1^{n\ b} - \eta^{a_1b}T_1^{n\ a_2} - \eta^{ba_2}T_1^{n\ a_1}\right)\,, \end{aligned} \tag{2.7}$$

$$\begin{aligned} M_1^{n\ b,a_1a_2} &= M^{n\ b,a_1a_2}x^2 - 2\left(M_1^{n\ b,a_2}x^{a_1} + M_1^{n\ b,a_1}x^{a_2}\right) - \\ &- \frac{2}{d-1}\left(2\eta^{a_1a_2}M_1^{n\ k,b}x_k - \eta^{a_1b}M_1^{n\ k,a_2}x_k - \eta^{ba_2}M_1^{n\ k,a_1}x_k\right) \end{aligned} \tag{2.8}$$

and

$$
\begin{aligned}
M^{n\ b_1b_2,a_1a_2} = {} & 2T^{na_1a_2}x^{b_1}x^{b_2} - T^{nb_1a_2}x^{a_1}x^{b_2} - T^{na_1b_2}x^{b_1}x^{a_2} + \\
& + 2T^{nb_1b_2}x^{a_1}x^{a_2} - T^{na_2b_2}x^{a_1}x^{b_1} - T^{na_1b_1}x^{a_2}x^{b_2} - \\
& - \frac{1}{d-1}\left(2\eta^{a_1a_2}K_1^{n\ b_1b_2} - \eta^{b_1a_2}K_1^{n\ a_1b_2} - \eta^{a_1b_2}K_1^{n\ b_1a_2} + \right. \\
& \qquad \left. + 2\eta^{b_1b_2}K_1^{n\ a_1a_2} - \eta^{a_2b_2}K_1^{n\ a_1b_1} - \eta^{a_1b_1}K_1^{n\ a_2b_2}\right) + \\
& + \frac{2}{d(d+1)}\left(2\eta^{a_1a_2}\eta^{b_1b_2} - \eta^{b_1a_2}\eta^{a_1b_2} - \eta^{a_1b_1}\eta^{b_2a_2}\right)T_2^n\,. \qquad (2.9)
\end{aligned}
$$

These currents have the following symmetry properties

$$
\begin{aligned}
K_2^{n\ a_1a_2} &= K_2^{n\ a_2a_1}\,, \\
K_1^{n\ a_1a_2} &= K_1^{n\ a_2a_1}\,, \\
M^{n\ b,a_1a_2} &= M^{n\ b,a_2a_1} = -M^{n\ a_1,a_2b} - M^{n\ a_2,a_1b}\,, \\
M_1^{n\ b,a_1a_2} &= M_1^{n\ b,a_2a_1} = -M_1^{n\ a_1,a_2b} - M_1^{n\ a_2,a_1b}\,, \\
M^{n\ b_1b_2,a_1a_2} &= M^{n\ b_1b_2,a_2a_1} = M^{n\ b_2b_1,a_1a_2}\,, \\
M^{n\ b_1b_2,a_1a_2} &+ M^{n\ b_1a_1,a_2b_2} + M^{n\ b_1a_2,b_2a_1} = 0\,. \qquad (2.10)
\end{aligned}
$$

Note that only the original current T^{abc} and the Lorentz-type currents (2.7) and (2.9) conserve in the nonconformal case considered in [4] (after adding appropriate terms containing the traces $T^{abc}\eta_{bc}$ that are not supposed to be zero in the non-conformal case). All other currents in the above list require the "stress tensor" T^{abc} to be traceless and generalize the dilatation and special conformal currents of the usual conformal case. From the analysis of this section it is seen that different currents classify according to their (i) symmetry properties, (ii) a degree of homogeneity in x^a and (iii) a highest power of x^2.

3. Conformal currents of any spin

Let us first present the full list of conserved higher spin currents for an arbitrary integer spin $s = \mathfrak{s} + 1$. Let $T^{a(s)}$ be totally symmetric and traceless, i.e.

$$T^{bca(s-2)}\eta_{bc} = 0\,. \qquad (3.1)$$

Provided that $T^{a(s)}$ is conserved,

$$\frac{\partial}{\partial x^n}T^{n\,a(\mathfrak{s})} = 0\,, \qquad (3.2)$$

one can construct a three-parametric family of the conserved tensors.

First one constructs a family of lower rank conserved totally symmetric traceless tensors

$$T_q^{a(s-q)} \stackrel{\mathrm{def}}{=} T^{a(s-q)b(q)}x_{b(q)}\,. \qquad (3.3)$$

Obviously, these tensors are symmetric and themselves satisfy the conditions (3.2), and (3.1). Therefore one can use the tensors $T_q^{a(s-q)}$ the same way as the original spin $s-q$ tensors $T^{a(s-q)}$.

The analysis of the spin 2 and spin 3 cases indicates another source of multiplicity of currents due to the possibility to multiply by powers of x^2. Taking into account that the conservation condition is homogeneous in x^a one looks for a current of the form

$$J^n = \sum_{\mathfrak{c}=0}^{\mathfrak{a}} (x^2)^{\mathfrak{a}-\mathfrak{c}} C(\mathfrak{a},\mathfrak{c},\mathfrak{s},\mathfrak{t}) T_{\mathfrak{c}}^{n\,a(\mathfrak{s}-\mathfrak{c})} \xi_{b(\mathfrak{t})\,,a(\mathfrak{s}-\mathfrak{c})b(\mathfrak{c})} x^{b(\mathfrak{t}+\mathfrak{c})} \tag{3.4}$$

with some coefficients $C(\mathfrak{a},\mathfrak{c},\mathfrak{s},\mathfrak{t})$ and arbitrary parameters $\xi_{b(\mathfrak{t})\,,a(\mathfrak{s})}$ corresponding to the traceless two-row Young diagram. An elementary computation shows that the current J^n conserves provided that

$$C(\mathfrak{a},\mathfrak{c},\mathfrak{s},\mathfrak{t}) = (-2)^{\mathfrak{c}} \frac{(\mathfrak{s}-\mathfrak{c}-\mathfrak{t})!}{\mathfrak{c}!(\mathfrak{a}-\mathfrak{c})!(\mathfrak{s}-\mathfrak{c})!} \tag{3.5}$$

and $\mathfrak{a}+\mathfrak{t} \le \mathfrak{s}$.

Taking into account that analogous formula for currents $T_{\mathfrak{b}}$ (3.3) also leads to the conserved currents one arrives at the final result

$$J^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(T|\xi) = \sum_{\mathfrak{c}=0}^{\mathfrak{a}} (-2)^{\mathfrak{c}} \frac{(\mathfrak{s}-\mathfrak{b}-\mathfrak{c}-\mathfrak{t})!}{\mathfrak{c}!(\mathfrak{a}-\mathfrak{c})!(\mathfrak{s}-\mathfrak{b}-\mathfrak{c})!} (x^2)^{\mathfrak{a}-\mathfrak{c}} T_{\mathfrak{b}+\mathfrak{c}}^{n\,a(\mathfrak{s}-\mathfrak{b}-\mathfrak{c})} \xi^{\mathfrak{a}}_{b(\mathfrak{t})\,,a(\mathfrak{s}-\mathfrak{b}-\mathfrak{c})b(\mathfrak{c})} x^{b(\mathfrak{t}+\mathfrak{c})} \tag{3.6}$$

for

$$\mathfrak{a}+\mathfrak{b}+\mathfrak{t} \le \mathfrak{s}\,. \tag{3.7}$$

Therefore, for any four non-negative integers $\mathfrak{a}$,$\mathfrak{b}$,$\mathfrak{s}$ and $\mathfrak{t}$ satisfying the inequality (3.7) there is an independent conserved current associated with the generalized stress tensor $T^{a(s)}$ and a two-row Young diagram parameter $\xi^{\mathfrak{a}}_{b(\mathfrak{t})\,,a(\mathfrak{s}-\mathfrak{b})}$. The label $\mathfrak{a}$ is introduced here to distinguish between parameters that belong to the same representation of the Lorentz group but are associated with different conserved currents.

The situation with half-integer higher spin supercurrents is analogous. Let $R^{a(\mathfrak{s}+1)}{}_{\alpha}$ be a totally symmetric conserved tensor-spinor of spin $s=\mathfrak{s}+3/2$ (α is the spinor index in d dimensions) satisfying the γ transversality condition, i.e.

$$\begin{aligned} \partial_a R^{a\,b(\mathfrak{s})}{}_{\beta} &= 0\,, \\ (\gamma_a)_{\alpha}{}^{\beta} R^{a\,b(\mathfrak{s})}{}_{\beta} &= 0 \end{aligned} \tag{3.8}$$

with the d-dimensional Dirac γ matrices

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab}\,. \tag{3.9}$$

Since the tensor-spinor $R^{a(\mathfrak{s}+1)}{}_\alpha$ is traceless as a consequence of (3.8), one can apply the bosonic construction (3.6) to build the conserved supercurrents

$$F^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(R|\xi) = \sum_{\mathfrak{c}=0}^{\mathfrak{a}} (-2)^{\mathfrak{c}} \frac{(\mathfrak{s}-\mathfrak{b}-\mathfrak{c}-\mathfrak{t})!}{\mathfrak{c}!(\mathfrak{a}-\mathfrak{c})!(\mathfrak{s}-\mathfrak{b}-\mathfrak{c})!} \times$$
$$\times\, (x^2)^{\mathfrak{a}-\mathfrak{c}} x^{b(\mathfrak{t}+\mathfrak{c})} \xi^{\mathfrak{b}}_{b(\mathfrak{t})\,,a(\mathfrak{s}-\mathfrak{b}-\mathfrak{c})b(\mathfrak{c})}{}^{\beta} R^{n\,a(\mathfrak{s}-\mathfrak{b}-\mathfrak{c})}_{\mathfrak{b}+\mathfrak{c}}{}_{\beta}\,, \tag{3.10}$$

where

$$R^{a(s-q)}_q \stackrel{\mathrm{def}}{=} R^{a(s-q)b(q)} x_{b(q)} \tag{3.11}$$

and the tensor-spinor parameter is irreducible

$$\xi^{\mathfrak{b}}_{b(u-1)a(1)\,,a(v)}{}^{\beta} = 0\,,$$
$$\xi^{\mathfrak{b}}_{b(u)\,,a(v)}{}^{\beta}(\gamma^a)_\beta{}^\alpha = 0\,. \tag{3.12}$$

Due to the γ-transversality condition (3.8) the multiplication of the right hand side of (3.10) by the factor of

$$\hat{x} = x^a\gamma_a \tag{3.13}$$

does not destroy the conservation property. This gives another set of conformal higher spin supercurrents

$$G^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(R|\xi) = \sum_{\mathfrak{c}=0}^{\mathfrak{a}} (-2)^{\mathfrak{c}} \frac{(\mathfrak{s}-\mathfrak{b}-\mathfrak{c}-\mathfrak{t})!}{\mathfrak{c}!(\mathfrak{a}-\mathfrak{c})!(\mathfrak{s}-\mathfrak{b}-\mathfrak{c})!} \times$$
$$\times\, (x^2)^{\mathfrak{a}-\mathfrak{c}} x^{b(\mathfrak{t}+\mathfrak{c})} \left(\xi^{\mathfrak{b}}_{b(\mathfrak{t})\,,a(\mathfrak{s}-\mathfrak{b}-\mathfrak{c})b(\mathfrak{c})} \hat{x} R^{n\,a(\mathfrak{s}-\mathfrak{b}-\mathfrak{c})}_{\mathfrak{b}+\mathfrak{c}}\right). \tag{3.14}$$

Note that the supercurrents F and G generalize the usual Q and S conformal supersymmetries.

The results of this paper rely on the fact of the existence of the traceless totally symmetric conserved tensors T or γ-transversal supercurrents R. The conserved traceless spin s tensors built in [3] from massless scalar and spinor matter fields have the form

$$T^{n(s)} = \sum_{k=0}^{s} (-1)^k \frac{(d/2-2)!(s+d/2-2)!}{k!(s-k)!(d/2+k-2)!(s+d/2-k-2)!} \times \tag{3.15}$$
$$\times \left[\partial^{n(k)}\varphi\,\partial^{n(s-k)}\varphi - \frac{k(s-k)}{d+2s-4}\,\eta^{n(2)}\partial^{n(k-1)}\,\partial_m\varphi\,\partial^{n(s-k-1)}\,\partial^m\varphi\right]\cdots$$

and

$$\mathcal{J}^{n(s)} = \sum_{k=0}^{s-1} \frac{(-1)^k(d/2-1)!(s+d/2-2)!}{k!(s-k-1)!(k+d/2-1)!(s+d/2-k-2)!} \times$$
$$\times \left[\partial^{n(k)}\bar{\psi}\,\gamma^n\,\partial^{n(s-k-1)}\psi - \frac{k(s-k-1)}{d+2s-4}\,\eta^{n(2)} \times\right.$$
$$\left.\times\, \partial_m\,\partial^{n(k-1)}\bar{\psi}\,\gamma^n\,\partial^m\,\partial^{n(s-k-2)}\psi\right]\cdots\,, \tag{3.16}$$

where

$$\partial^{a(p)} = \partial^{a_1} \dots \partial^{a_p} \,, \tag{3.17}$$

and dots stand for discarded terms containing more than one factor of $\eta^{n(2)}$ (such terms do not contribute to (3.6) because the parameter $\xi_{b(\mathfrak{t}),a(\mathfrak{s})}$ is traceless).

The half-integer spin s gamma-traceless conserved tensor-spinor has the form

$$\begin{aligned} R^{a(s-1/2)} = \sum_{l,n,p=0}^{\infty} (-1)^l \frac{(d/2+p+l+n-1)!}{l!n!p!2^p(d/2+p+l-1)!(d/2+p+n-1)!} \times \\ \times \Bigg[\left(\frac{d}{2}+p+n-1\right) \delta\left(s-\frac{1}{2}-l-n-2p\right) \eta^{a(2p)} \partial^{a(l)} \partial^{b(p)} \psi \partial^{a(n)} \partial_{b(p)} \varphi - \\ -\frac{1}{2}\delta\left(s-\frac{1}{2}-l-n-2p-1\right) \eta^{a(2p)} \gamma^{a(1)} \partial^{a(l)} \partial^{b(p)} \gamma^{b(1)} \psi \partial^{a(n)} \partial_{b(p+1)} \varphi \Bigg], \end{aligned} \tag{3.18}$$

where, for odd d, the factorials have to be understood as the corresponding Γ functions, $\delta(0) = 1$ and $\delta(n) = 0$ for $n \neq 0$, and we use the shorthand notation

$$\eta^{a(2p)} = \underbrace{\eta^{a(2)} \dots \eta^{a(2)}}_{p} \,. \tag{3.19}$$

Note that the sum in (3.18) contains a finite number of terms due to the factors of δ. The proof of the facts that the supecurrent (3.18) is $\gamma-$transversal and conserves as a consequence of the field equations for the massless spinor and scalar

$$\begin{aligned} \Box\varphi &= 0 \,, \\ \hat{\partial}\psi &= 0 \,, \\ \Box &\equiv \partial_b \partial^b \,, \\ \hat{\partial} &\equiv \partial_b \gamma^b \end{aligned} \tag{3.20}$$

is straightforward but a little tedious. To the best of our knowledge the constructed conformal higher spin supercurrents are new. We therefore reproduce here their complete form (i.e. keeping all higher traces).

Let us note that the form of the irreducible currents (3.15), (3.16) and (3.18) and, therefore, (3.6), (3.10) and (3.14) is fixed unambiguously by the conservation and irreducibility conditions provided that there is no explicit dependence on x^m and an order of derivatives is fixed to be minimal for a given rank of the current. (This is not the case for the usual (non-conformal) currents that are not required to be traceless or γ-transversal.) Relaxing the latter condition one constructs more conserved currents. For example, the following currents are obviously conserved and irreducible:

$$\Box T^{n(s)} \,, \qquad x^m \partial_m T^{n(s)} \,, \qquad \hat{\partial} R^{a(s-1/2)} \,. \tag{3.21}$$

A less trivial example is

$$T^{n(s+1)} = \partial^{n(1)} T^{n(s)} + \frac{1}{s+d-1} x^m \partial_m \partial^{n(1)} T^{n(s)} + \\ + \frac{s}{(s+d-1)(d+2s-3)} \left(\eta^{n(2)} \Box - \partial^{n(2)} \right) \left(T^{n(s-1)m} x_m \right) . \qquad (3.22)$$

However, we conjecture that all the conserved successors containing derivatives of the original currents do not give rise to new charges, thus describing various improvements of the original currents.

Indeed, the conserved currents give rise to the conserved charges in the standard way by virtue of the integration over a space-like surface of co-dimension 1

$$Q_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(T|\xi) = \int_{M_{d-1}} \tilde{J}_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b},}(T|\xi) \,, \qquad (3.23)$$

where $\tilde{J}_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(T|\xi)$, is the on-mass-shell exact $d-1$ form dual to $J^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(T|\xi)$, i.e.

$$J^{n_1}_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(T|\xi) = \frac{1}{(d-1)!} \epsilon^{n_1 \ldots n_d} \tilde{J}_{n_2 \ldots n_d |\, \mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(T|\xi) \,. \qquad (3.24)$$

Therefore, charges identify with the current cohomologies. The trivial class of exact forms $\tilde{J}_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(T|\xi)$ describe various "improvements" of the currents. In terms of the original vector currents this equivalence has the standard form

$$J^n \sim J^n + \partial_m H^{nm} \qquad (3.25)$$

for any antisymmetric bivector $H^{nm} = -H^{mn}$ locally expressed in terms of the original fields. A straightforward analysis then shows that the following equivalences take place

$$J^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(\Box T|\xi) \sim 2(d+2\mathfrak{a}+2\mathfrak{t}-4) J^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a}-1,\mathfrak{b}}(T|\xi) \,, \qquad (3.26)$$

and

$$F^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(\hat{\partial} R|\xi) \sim -2 G^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a}-1,\mathfrak{b}}(R|\xi) \,, \\ G^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(\hat{\partial} R|\xi) \sim -(d+2\mathfrak{a}+2\mathfrak{t}-2) F^n_{\mathfrak{s},\mathfrak{t},\mathfrak{a},\mathfrak{b}}(R|\xi) \qquad (3.27)$$

(with the convention that the currents with negative $\mathfrak{a}$ are equal to zero).

4. AdS/CFT correspondence

The parameters $\xi^{\mathfrak{a}}_{b(\mathfrak{t}),a(\mathfrak{s}-\mathfrak{b})}$ used in (3.6), (3.10) and (3.14) as projectors to the irreducible components of the currents can in fact be identified with the higher spin global symmetry parameters. This is most obvious from the relationship between the currents and conserved charges (3.23). Therefore the conformal (super)currents found in the section 3 suggest a pattern of the global symmetry parameters of a yet unknown conformal higher spin symmetry algebra in any dimension. Analogously, the non-conformal (i.e. AdS or Poincare) currents found in [4] suggest a pattern of the AdS higher spin algebra in d dimensions.[2] For the particular cases of $d = 3$ or 4 in which the higher spin symmetries are known explicitly it was demonstrated in [4] that indeed the one-to-one correspondence between the patterns of the higher spin currents and AdS higher spin symmetries takes place. Here we perform another check of the consistency of the pattern of the higher spin currents from the perspective of the AdS/CFT correspondence.

It is well known that the d-dimensional conformal algebra $o(d, 2)$ is isomorphic to the d+1 dimensional anti-de Sitter algebra. The dynamical realization of this identity has been achieved in the framework of the AdS/CFT correspondence between field theories in the AdS space and conformal models at the boundary of the AdS space [7]. Let us show that the structure of the currents obtained indicates that the analogous correspondence is true for the higher spin symmetries.

Indeed, as shown in the section 3, conformal higher spin symmetry parameters are irreducible tensors or tensor-spinors $\xi^{\mathfrak{a}}_{b(\mathfrak{t})\,,a(\mathfrak{s}-\mathfrak{b})}$ with $\mathfrak{s} = s - 1$ for integer spins s and $\mathfrak{s} = s-3/2$ for half-integer spins s, and arbitrary non-negative integer parameters $\mathfrak{a}, \mathfrak{b}, \mathfrak{t}$ satisfying the restriction $\mathfrak{a} + \mathfrak{b} + \mathfrak{t} \leq \mathfrak{s}$. The integers $\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{t}$ parametrize different conformal symmetries associated with the spin s. On the other hand, as shown in [4], usual (i.e. non-conformal) higher spin symmetry parameters $\Xi_{m(\mathfrak{t}),n(\mathfrak{s})}$ correspond to all irreducible two-row (tensor or tensor-spinor) Young diagrams, for a given spin s every diagram appears once ($0 \leq \mathfrak{t} \leq \mathfrak{s}$).

Let us now assume that the parameters $\Xi_{\hat{m}(\mathfrak{t}),\hat{n}(\mathfrak{s})}$ describe AdS higher spin symmetry in $d + 1$ dimensions, i.e. $\hat{m}, \hat{n} = 0 \div d$, and analyze what is a result of the dimensional reduction of this set of parameters to d dimensions. The conclusion is that the pattern of the dimensionally reduced AdS_{d+1} parameters is exactly the same as the set of the conformal higher spin parameters $\xi^{\mathfrak{a}}_{b(\mathfrak{t})\,,a(\mathfrak{s}-\mathfrak{b})}$ with allowed values of $\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{t}$. To see this it is enough to observe that for the irreducible representations of the orthogonal algebras the following reduction rule is true [6]

$$\Xi_{\hat{m}(\mathfrak{t}),\hat{n}(\mathfrak{s})} \rightarrow \sum_{\mathfrak{b}=0}^{\mathfrak{s}-\mathfrak{t}} \sum_{\mathfrak{c}=0}^{\mathfrak{t}} \oplus \xi_{b(\mathfrak{t}-\mathfrak{c})\,,a(\mathfrak{s}-\mathfrak{b})} \tag{4.1}$$

both for (traceless) tensors and ($\gamma-$transversal) tensor-spinors. As a result, the set of the representations of the $d + 1$-dimensional Lorentz algebra, associated with the higher spin AdS_{d+1} parameters $\Xi_{\hat{m}(\mathfrak{t}),\hat{n}(\mathfrak{s})}$, exactly reproduces the set of the representations of the d-dimensional Lorentz algebra associated with the conformal higher spin parameters. This is a strong indication that AdS/CFT correspondence extends to the (global) higher spin symmetries.

[2]Since from the $d = 4$ example it is known (see [4] and references therein) that it is AdS geometry rather than the Poincare one that is adequate to the higher spin dynamics at the interaction level, the former possibility is most interesting.

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Finally, let us note that the relationship (4.1) applied to one dimension higher tells us that the pattern of the AdS higher spin parameters $\Xi_{m(\mathfrak{t}),n(\mathfrak{s})}$ corresponds to the dimensional reduction of the irreducible representation $\Xi_{M(\mathfrak{s}),N(\mathfrak{s})}$ of the algebra $o(d-1,2)$ $(N, M = 0, \ldots, d+1)$ described by the two-row rectangular (traceless tensor or $\gamma-$transversal tensor-spinor) Young diagram of the length $\mathfrak{s}$. The same is therefore true for the higher spin conformal symmetry parameters as resulting from the dimensional reduction of $\Xi_{M(\mathfrak{s}),N(\mathfrak{s})}$ to two dimensions less. This result is a natural generalization of the trivial fact that the usual AdS and conformal symmetry parameters span the adjoint representation of the respective orthogonal algebras, thus corresponding to the antisymmetric parameter $\Xi_{M,N}$, i.e. $\mathfrak{s} = 1$.

To summarize, the higher spin extension of $o(p,2)$ decomposes into the irreducible representations of $o(p,2)$ described by the two-row rectangular Young diagrams independently on whether $o(p,2)$ is interpreted as AdS algebra (i.e. $p = d-1$) or conformal one (i.e. $p = d$). This conclusion may be important for the analysis of the higher spin theories in higher dimensions.

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References

[1] D.M. Lipkin, *Existence of a new conservation law in electromagnetic theory*, *J. Math. Phys.* **5** (1964) 696;
T.A. Morgan, *Two classes of new conservation laws for electromagnetic field and other massless fields*, *J. Math. Phys.* **5** (1964) 1659;
D.A. Fradkin, *Conserved quantities associated with symmetry transformations of relativistic free-particle equations of motion*, *J. Math. Phys.* **6** (1965) 879;
T.W.B. Kibble, *Conservation laws for free fields*, *J. Math. Phys.* **6** (1965) 1022;
R.F. O'Connel and D.R. Tompkins, *Generalized solutions for massless free fields and consequent generalized conservation laws*, *J. Math. Phys.* **6** (1965) 1952.

[2] F.A. Berends, G.J. Burgers and H. van Dam, *Explicit construction of conserved currents for massless fields of arbitrary spin*, *Nucl. Phys.* **B 271** (1986) 429;
D. Anselmi, *Theory of higher spin tensor currents and central charges*, *Nucl. Phys.* **B 541** (1999) 323 [`hep-th/9808004`].

[3] D. Anselmi, *Higher-spin current multiplets in operator-product expansions*, *Class. and Quant. Grav.* **17** (2000) 1383 [`hep-th/9906167`].

[4] M.A. Vasiliev, *Higher spin gauge theories: star-product and AdS space*, in *Golfand's memorial volume. Many faces of the superworld*, M. Shifman ed., World Scientific Publishing, Singapore 2000 [`hep-th/9910096`].

[5] M.A. Vasiliev, *Free massless fields of arbitrary spin in the de Sitter space and initial data for a higher spin superalgebra*, *Fortschr. Phys.* **35** (1987) 741.

[6] A.O. Barut and R. Rączka, *Theory of group representations and applications*, PWN, Warszawa 1986.

[7] P.A.M. Dirac, *A remarkable respresentation of the* $3+2$ *de Sitter group*, *J. Math. Phys.* **4** (1963) 901;
J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200];
S. Ferrara and C. Fronsdal, *Conformal Maxwell theory as a singleton field theory on* AdS_5*, IIB three branes and duality*, *Class. and Quant. Grav.* **15** (1998) 2153 [hep-th/9712239];
M. Gunaydin and D. Minic, *Singletons, doubletons and M-theory*, *Nucl. Phys.* **B 523** (1998) 145 [hep-th/9802047];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

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